

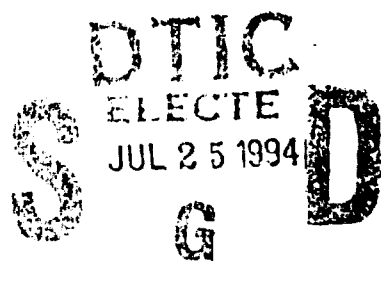
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A RAND NOTE

Experiments In Variable-Resolution
Combat Modeling

Richard J. Hillestad, John G. Owen,
Donald Blumenthal



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Prepared for the
Defense Advanced Research Projects Agency

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PREFACE

This Note was prepared for the Defense Advanced Research Projects Agency. The work was performed in the Applied Science and Technology program of RAND's National Defense Research Institute (NDRI), a federally funded research and development center supported by the Office of the Secretary of Defense and the Joint Staff. As one of a trilogy of documents working through simple examples to illustrate deeper issues that arise in variable-resolution modeling, this Note describes a set of experiments comparing combat models with different levels of resolution. The other documents are Richard J. Hillestad and Mario Juncosa, *Cutting Some Trees to See the Forest: On Aggregation and Disaggregation in Combat Models*, MR-189-DARPA, 1993, and Paul Davis, *An Introduction to Variable-Resolution Modeling and Cross-Resolution Model Connection*, R-4252-DARPA, 1993. Initial versions of the three documents were presented as papers at a conference on variable-resolution modeling organized by RAND and the University of Arizona and sponsored by DARPA and the Defense Modeling and Simulation Office, in May 1992.

John G. Owen, currently at RAND, is a researcher from the UK Defence Operational Analysis Centre, and Donald Blumenthal is with Lawrence Livermore National Laboratory at Livermore, California.

SUMMARY

This Note examines the differences in combat outcomes predicted by models of different resolution applied to identical combat situations. First, hypothetical combat situations are posed, then several models of varying degrees of resolution in the spatial representation, aggregation of forces, and time step are used to predict losses and battle winners. Both stochastic and deterministic simulations are used. Comparison of outcomes provides important insights into the problems of aggregation. Observations from this set of experiments are as follows. Intuition regarding outcomes, causes, and effects is frequently wrong, leading to bad approximations in the aggregate. Scaling for different levels of resolution is possible, but a method of predicting the appropriate scaling technique and factors has not been found. The differences in outcomes between stochastic and deterministic models are most pronounced in the "fair-fight" regime, in which the force balance (accounting for situational factors) is almost even. Because defense analysis frequently operates in this regime (getting "just enough" force to a theater or because constrained defense budget allocations may not permit overwhelming odds), this implies that great care should be taken to understand the possible variance in outcomes.

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1. INTRODUCTION

Aggregate, low-resolution combat models are needed for a number of reasons. The fact that much historical data is of an aggregate nature without the details to totally reconstruct battles means that models based on such data must either operate at an aggregate level of resolution consistent with the data or the modeler/analyst/gamer is forced to guess at interactions, such as fire allocation, detailed acquisition predictions, and small-unit objectives. Command and control decisions frequently require information at an aggregate level: The commander or his intelligence branch estimates the "strength" of opposing and friendly forces to decide on the commitment of reserves and when to move. Aggregation is often necessary in analysis to comprehend and explain phenomena (the forest-vs.-trees argument). Other issues such as cost, the need to produce results to meet deadlines, and repeatability also force the analyst away from detailed weapon-on-weapon analysis. At the same time, such aggregations should not be done arbitrarily. Unfortunately, there is very little theory and science in most approaches that have been taken to aggregation in combat modeling.¹ In a companion report,² some of the theoretical issues involved in aggregation and disaggregation are examined. In this Note, we look at the problem more empirically, comparing the results of simulations at various levels of aggregation.

Our approach was to perform a controlled set of simulation experiments on the same combat problem using simulations with differing resolutions. The simulation results reported here are derived from two basic models. The first model, a detailed weapon-on-weapon, event-stepped, stochastic model, simulated each individual firing decision and round fired. The second model, a time-stepped, deterministic spreadsheet, aggregated fighting units into company, battalion, and regiment groups. Using the two models, we compared results when changing the spatial resolution, configuration,³ and object (unit) resolution. We also examined the differences between results simulated deterministically and those

¹Davis, Paul K., and Donald Blumenthal, *The Base-of-Sand Problem: A White Paper on the State of Military Combat Modeling*, RAND, N-3148-OSD/DARPA, 1991.

²Hillestad, Richard J., and Mario L. Juncosa, *Cutting Some Trees to See the Forest: On Aggregation and Disaggregation in Combat Models*, RAND, MR-189-DARPA, 1993.

³See Horrigan, Timothy, *Configurational Theory and the Mathematical Modeling of Combat: Realizing the Potential of Models and Simulations of Combat to Improve the Effectiveness of Weapons, Tactics, and Training*, Horrigan Analytics, HAS 91-17-1, 1991, for another discussion of the problems related to configuration and aggregation.

obtained with the stochastic model. The following sections describe the experimental frame, models used, results, possible approaches to developing consistent aggregations, and general observations drawn from the research.

2. DETAILED STOCHASTIC SIMULATION

The first model used is detailed (high resolution) in that it represents the individual systems and shots of those systems. On the other hand, it was used simply in the analysis in that it did not consider acquisition, and movement was constrained to straight paths on flat terrain with no obstacles. Each individual tank or other vehicle is described in terms of its side, type, position, speed and direction of movement, number of rounds remaining, and state—alive or dead. Systems are configured in groups, each of which moves along a straight path until an objective line is reached.

The ground is assumed to be flat and free of obstacles. All vehicles can always be seen, with no delay for searching. A dead vehicle may not be recognized as such until a given time after it is killed, or until a given number of hits have been scored on it. The perceived state of a vehicle is the same for all observers.

In the model, each system always fires at the closest enemy in range that is perceived to be alive at the moment of firing. Each system type has a rate of fire; the time between shots is not affected by the need to switch targets. Furthermore, effectiveness and accuracy are not affected by the motion of the firer or target. Probability of kill is range dependent, as is time of flight. Projectiles are assumed to be unguided after launch, or fire-and-forget; the projectile time of flight is not added to the time to the firer's next shot. When the projectile reaches the target, a random number is compared with the single-shot kill probability (P_k) to determine whether a kill has occurred; any delay in perceiving a kill is measured from this time.

Because of the stochastic nature of the model, the battle is repeated a number of times. There are two types of output: statistical, showing the cumulative results over all replications, and graphical, showing the movements and fate of systems in one selected replication.

The model is implemented in the MODSIM object-oriented simulation language.¹

¹Anyone wishing to know more about this model should contact R. Hillestad or J. Owen at RAND.

3. THE SCENARIO AND RESULTS OF DETAILED SIMULATION

Figure 1 shows the beginning of the main scenario used in this analysis. The grid lines are at a 500-m spacing in both axes, but in order to fit the scenario on the display, different scale factors have been used for the x- and y-directions. The 33 tanks of the 3 defending companies are shown individually. The attacking tanks (99 in number) are also individually marked, but because they are closer together, the symbols overlap at this scale. The attackers are organized into 3 battalions, 2 forward and 1 back, each composed of 3 companies, 2 forward and 1 back.

Figure 2 shows the simplified probability-of-kill curves used in the scenario. Comparing this figure with the initial positions in Figure 1, we see that all companies are initially out of range of the enemy. The attacker will move all companies in the formation forward simultaneously, at a constant speed. In the simulation, as systems are killed, they stop on the battlefield. When they have taken a certain number of shots, or after a specified time has passed, they are perceived by *all* other systems on the battlefield to be dead. Up until that time they can draw fire. In the initial cases, we have set the perception-delay time to zero so that a system is instantaneously perceived to be dead when it is killed. Later cases will show the result of the perception delay.

The scenario gives a 3-to-1 numerical advantage to the attacker and a range/ P_k advantage to the defender. The 3:1 rule¹ suggests that the outcome in this battle should favor neither the attacker nor the defender. As we show, this is far from true. Figure 3 shows snapshots of the battle at two later stages in one of 30 replications run. The attacker is closing in on the defender's position, at 30 kilometers per hour (km/hr). The defender has a 1-km range advantage, and is using it. The top diagram shows that the attacker's forward companies have already been decimated—the unfilled symbols are dead attackers—whereas the attacker is not yet in range to retaliate. The bottom diagram shows the end of this battle. The attacker has been wiped out at no loss to the defender; indeed, the attacker barely manages to get into range to fire a few shots.

This outcome is explained by the curve in Figure 4, which was derived from the P_k -vs.-range curves in Figure 2. In the region from 3 to 4 km, the defender can shoot at the attacker with some effectiveness but the attacker cannot return fire with any effectiveness.

¹See the debate about this rule in Mearsheimer, John J., "Assessing the Conventional Balance: The 3:1 Rule and Its Critics," *International Security*, Spring 1989, and in Epstein, Joshua M., "The 3:1 Rule, the Adaptive Dynamic Model, and the Future of Security Studies," *International Security*, Spring 1989.

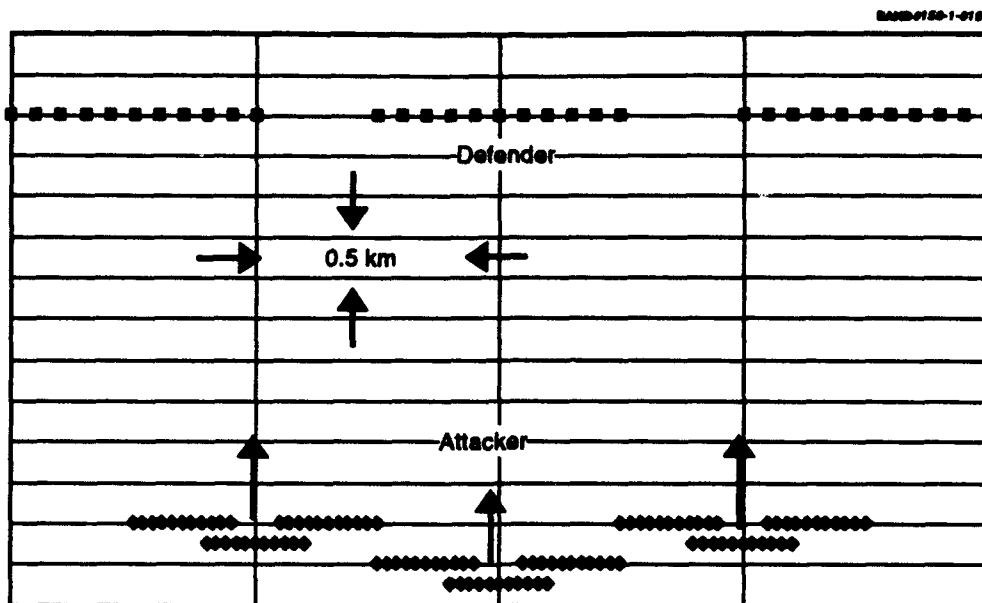


Figure 1—Initial Positions in Scenario

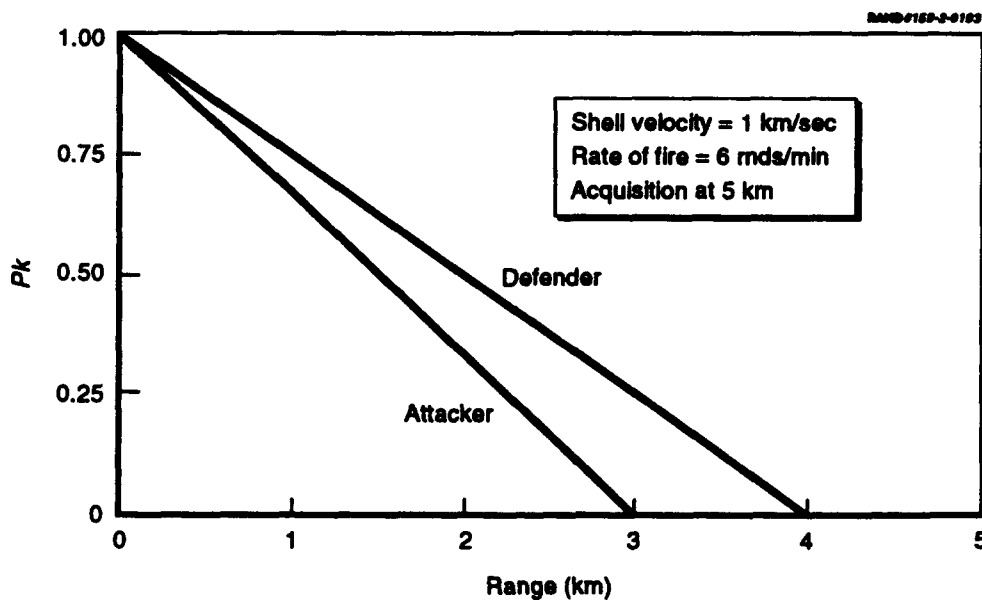
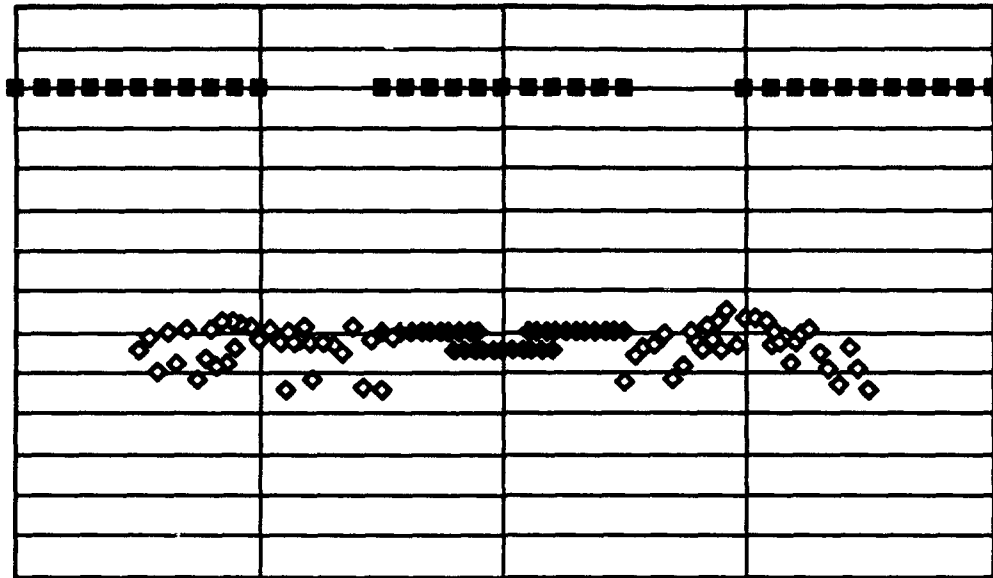
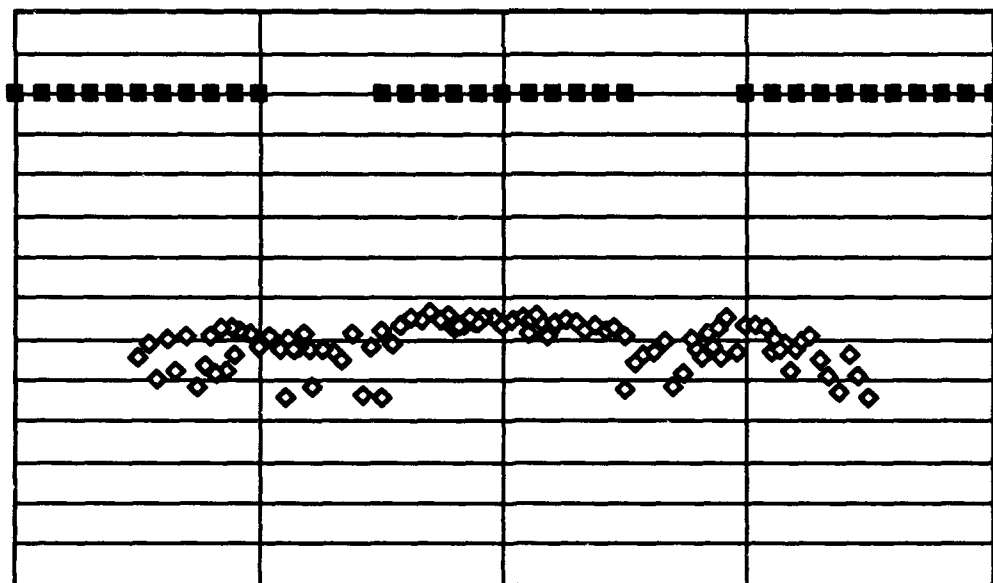


Figure 2—Simplified Probability-of-Kill Curves

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State after 5 min



State after 7 min: end of battle

Figure 3—Battle Results at an Attacker Closing Speed of 30 km/hr

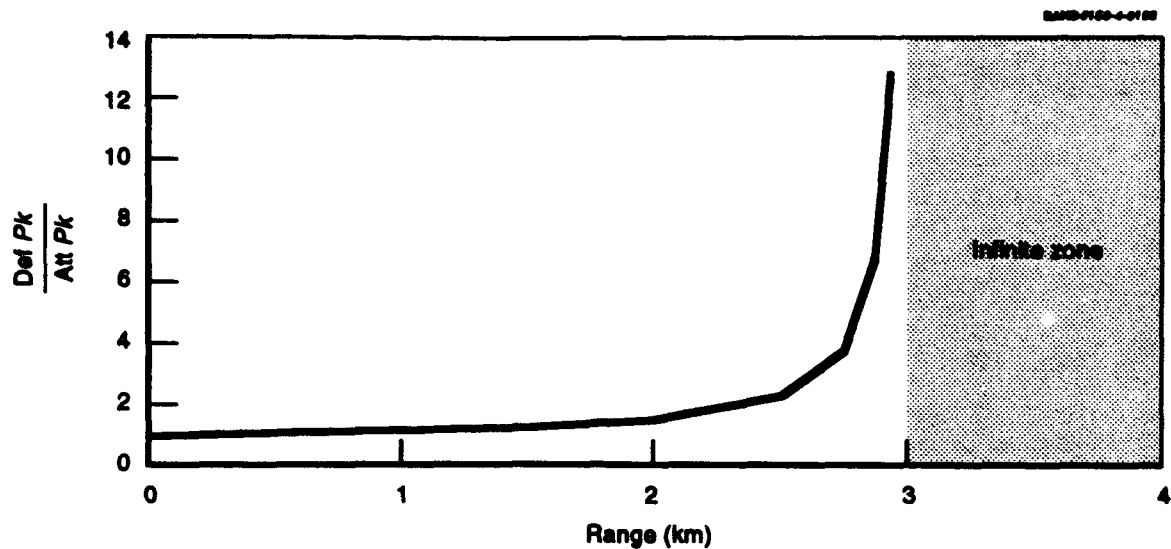


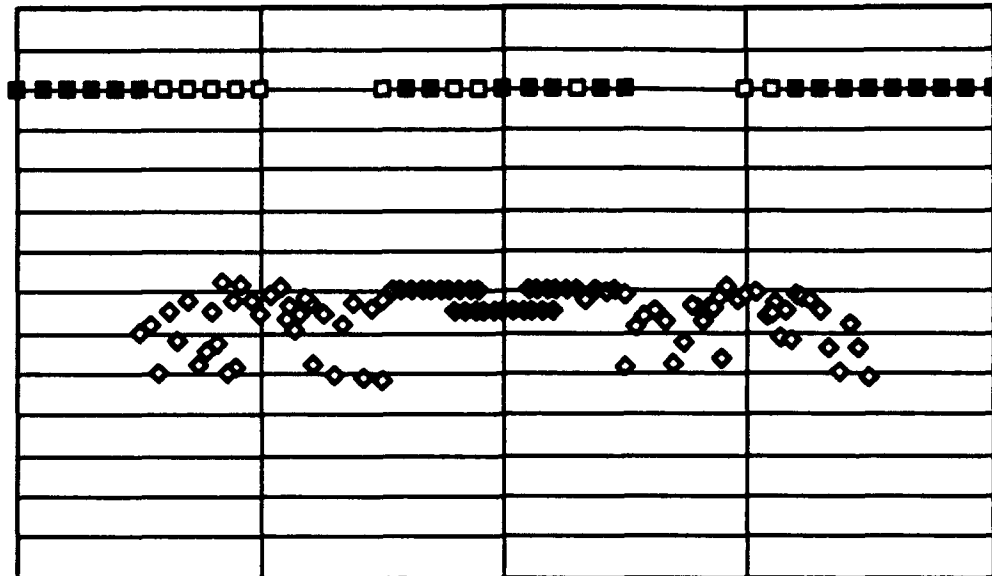
Figure 4—Defender/Attacker P_k Ratio as a Function of Range

At less than 3 km, the ratio of P_k advantage decreasingly favors the defender. The attacker does have a numerical advantage, so that once in range the advantage quickly switches to the attacker.

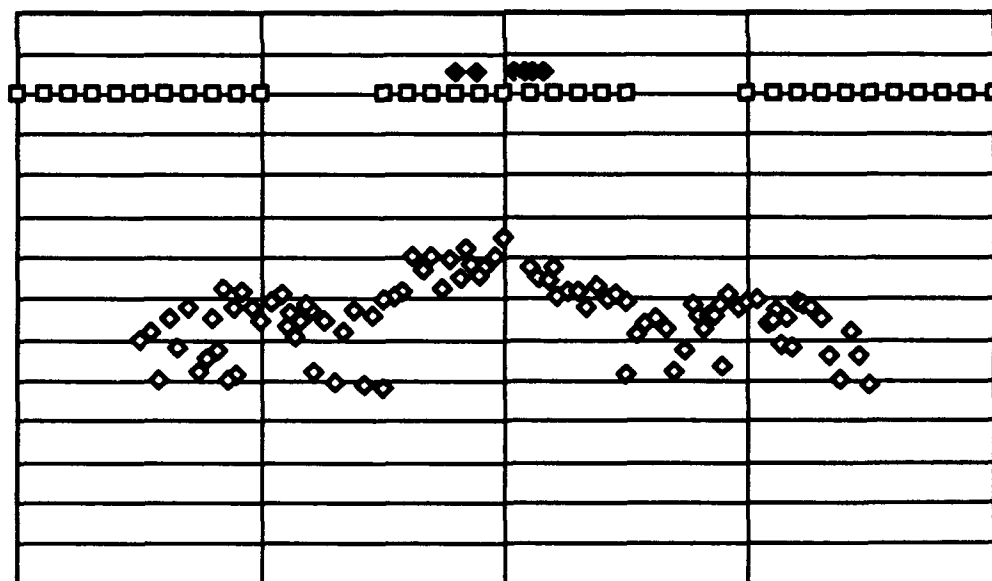
The problem for the attacker is to cross the "gauntlet" of defender fire and get into his effective fire zone before the defender defeats him. The problem for the defender is to destroy enough of the attacker before he can get into range and use his numerical advantage. In the case shown, the attacker crosses the zone of infinite defender advantage too slowly, so that all systems are destroyed before they get into range. It is relatively easy to calculate this outcome. At 30 km/hr it takes an attacker system 2 min to cross the zone between 3 and 4 km. The attacker battalions are spread 1 km in depth, as well, so that it takes about 4 min for all attacker systems to cross the zone. In 4 min, the defender can fire $33(\text{systems}) \times 6(\text{shots/min}) \times 4(\text{min}) = 792$ shots. The average P_k in the zone is 0.125, and the expected number of kills as the attacker crosses the zone is $0.125(\text{kills/shot}) \times 792(\text{shots}) = 99$ kills.

One option for the attacker is to get across the disadvantageous zone faster. Figure 5 shows a run with the same initial deployments but with a greater attacker speed of 45 km/hr. As stated earlier, the greater speed has no effect on either side's gunnery performance. At the intermediate stage, after 4 min, the forward attacker battalions have been wiped out, but not before getting far enough to do some damage to the defender. By the end of the battle, after 8 min in this specific replication, the defender has been wiped out, and a few surviving attackers are past the defender's position. The higher speed has allowed the attacker to cross the zone of defender advantage fast enough to win the battle, although it is something

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State after 4 min



State after 8 min; defender wiped out by 6 min

Figure 5—Battle Results at an Attacker Closing Speed of 45 km/hr

of a Pyrrhic victory. We have not considered the implications such large losses would have for either side's willingness to continue in the battle.

To further illustrate the point about the attacker's needing to get across the lethal zone, we varied the initial attacker configuration. Figure 6 shows an attacker deployment with all 3 battalions forward. The attack is therefore more concentrated in an attempt to get more weapons into range faster. The 4-min, intermediate snapshot in Figure 7 shows that if this formation advances at 45 km/hr, the lead companies of each battalion suffer heavily, but the defender also takes losses. The attacker wipes out the defense in this run, with a greater number of survivors than with a 2-up, 1-back formation at the same speed.

To test the model for structural bias that might favor one side or the other, we also created a meeting-engagement scenario: a completely symmetrical battle between Red and Blue. There is no range advantage to either side; both use the "Attacker" P_k curve of Figure 2. Figure 8 shows the initial positions in the battle; each side moves forward to attack the other from these positions. In Figure 9 the intermediate snapshot shows each side's forward companies almost wiped out. By the end of the battle, in this particular case, by roll of the dice, Blue is left with 6 survivors out of 99. We will not be showing any more results for this scenario, but it has been run for 30 replications, resulting in 14 Blue wins and 16 Red wins—a *win* being defined as having at least 1 survivor at the end of the battle. Other statistics gathered from these replications indicate the model does not have apparent biases.

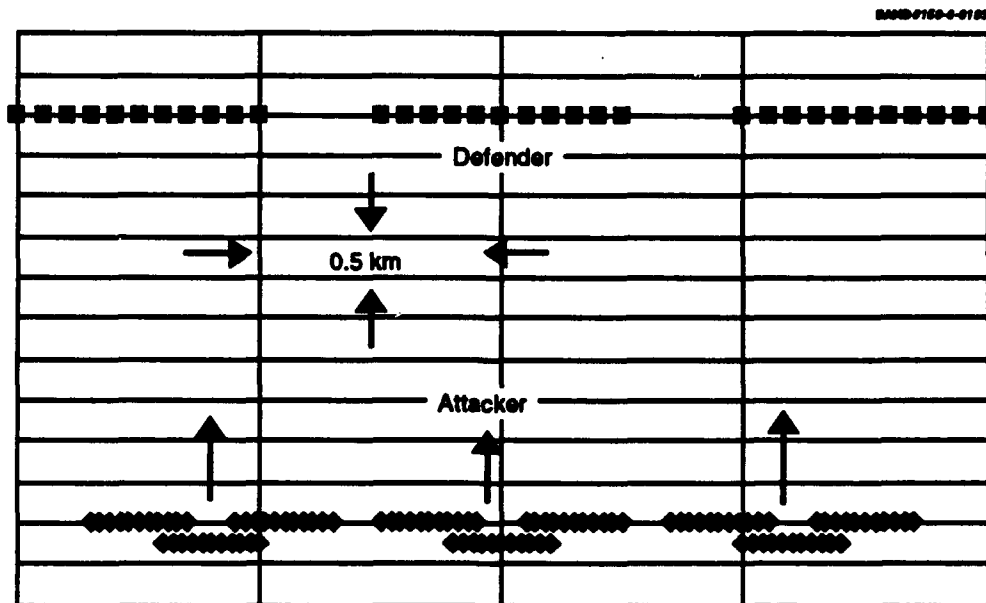
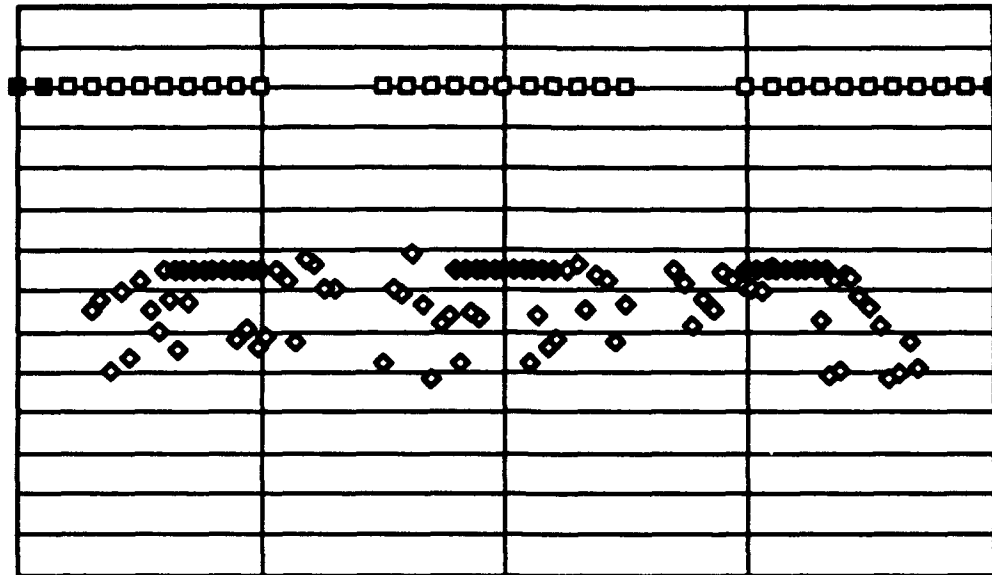
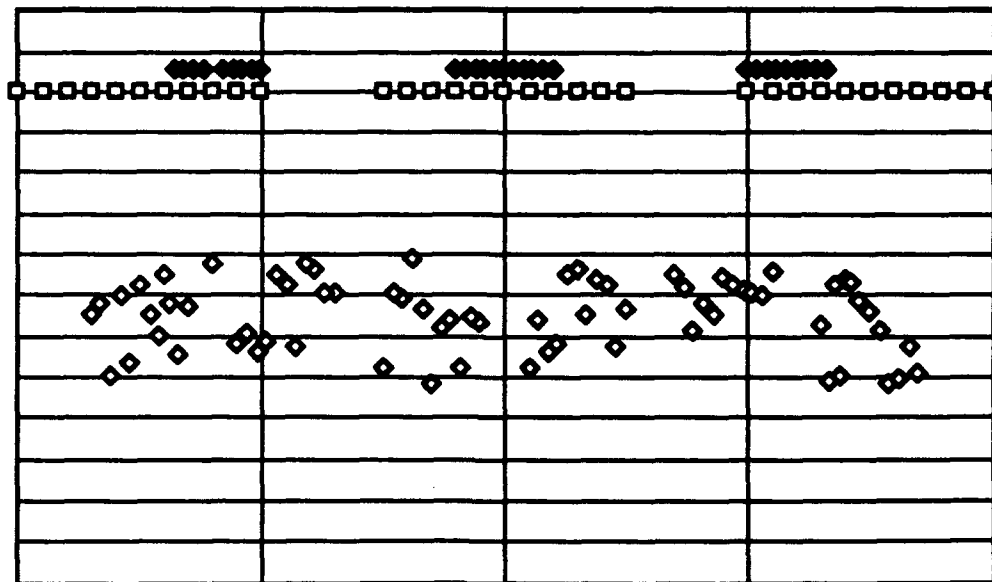


Figure 6—Initial Position: All Attacker Battalions Forward

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State after 4 min



State after 8 min; defender wiped out by 5 min

Figure 7—Battle Results at 45 km/hr and All Attacker Battalions Forward

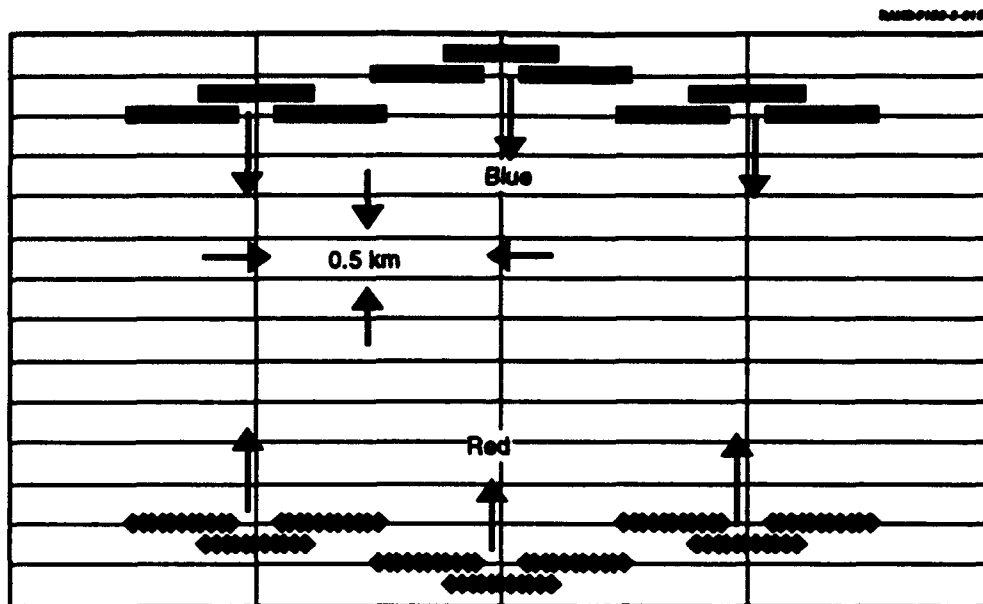


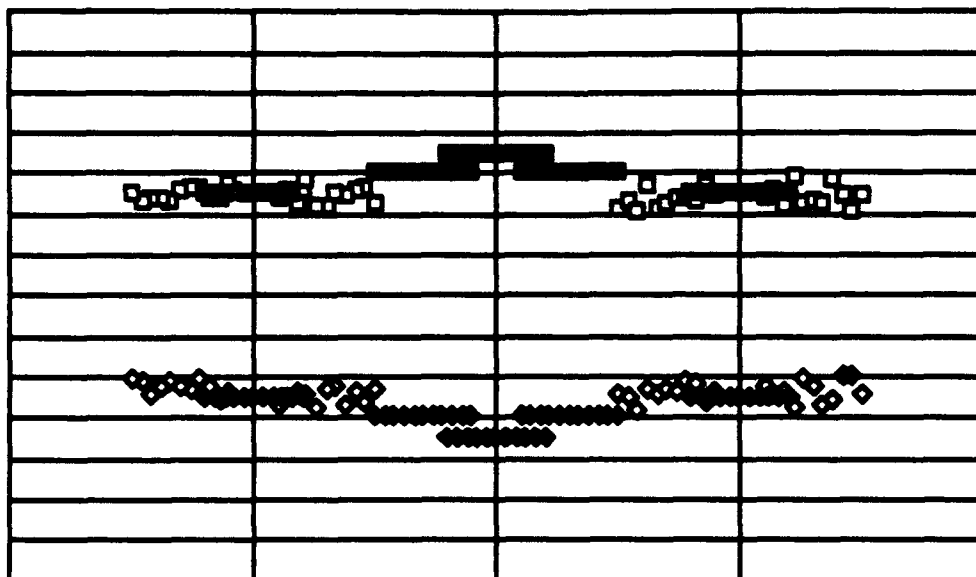
Figure 8—Initial Positions in a Meeting Engagement

The number of survivors on the winning side ranged from 3 to 37. We discuss the implications of such a large variance in survivors in Section 6.

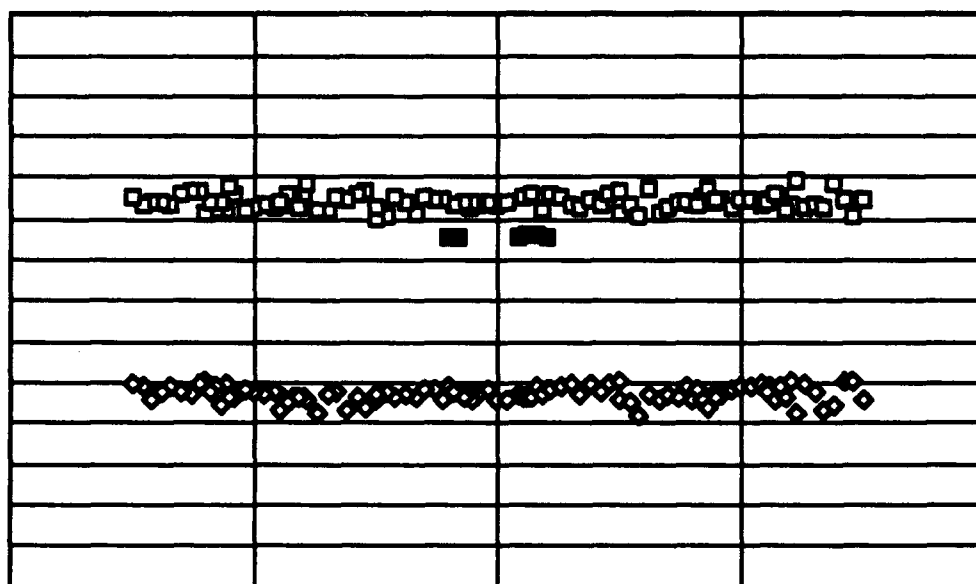
The previous battlefield graphic displays showed single replications; the positions in those replications depend on the particular values drawn for random numbers. Figure 10 summarizes statistical results for the initial scenario. The quantities plotted here and in the figures that follow are the number of defender and attacker tanks surviving at the end of the battle, where the battle is always fought to the annihilation of one side or the other. The results are for 33 defending tanks against 99 attacking tanks, run for 30 replications. The shaded part of the column shows one standard deviation about the mean. Above each column is shown the number of replications (out of 30) won by defender and attacker, where *winning* means having at least 1 tank left. Many of these victories are in fact Pyrrhic.

In the figure the effects of changing the speed with which the attacker closes on the defender's position are shown. At 15 km/hr the defender wins in all replications, without losing a single tank; the attacker is not moving fast enough to cross the zone where the defender can fire but he cannot. At 30 km/hr, the defender always wins, but the attacker occasionally gets close enough to do a little damage. At 45 km/hr, there is a wide variability in the number of survivors. The defender wins 24 out of 30 replications, but takes heavy losses. When the attacker wins, he has few survivors. At higher speeds, the advantage swings decisively to the attacker, although he always takes substantial losses. One

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State after 6 min



State after 10 min; one side wiped out by 9 min

Figure 9—Battle Results for a Meeting Engagement

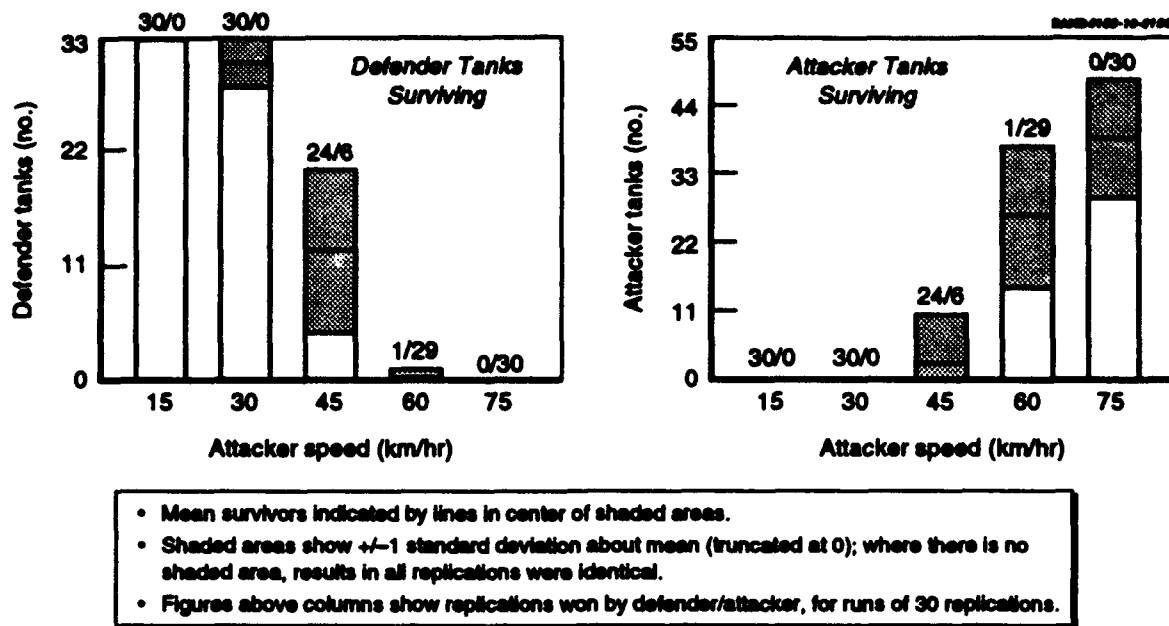


Figure 10—Stochastic Model Results: Effect of Attacker Speed

observation is that the variance is largest when the fight is "fair" or nearly equal. As should be expected, when one side or the other has a predominance of force, the variance in outcome is relatively small. This type of result has been reported elsewhere.²

All the results thus far have assumed perfect *perception*: When a tank was destroyed, it was immediately known by all enemy tanks. Figure 11 shows the effects of varying perception. All results are for a speed of advance of 45 km/hr.

The first case is with perfect perception, as in the previous figure. In the second case, there is a 10-sec delay following a kill of a tank on either side, before the enemy realizes that it is dead. Shots will therefore be wasted on dead targets, favoring the attacker: In the early stages of battle many of the forward attackers are killed, and they draw fire away from those behind or beside them, allowing more attackers to move into firing range. Note that the attacker/defender "win" ratio switches dramatically with a 10-sec delay: The defender cannot afford to waste these shots as the attacker moves across the zone in which the defender has an advantage.

²Hofmann, Hans W., "On an Approach to Stochastic Modeling of Combat at the Corps/Army Level," Paper presented at the TIMS/ORSA/MAS Meeting, Nashville, Tennessee, May 1991.

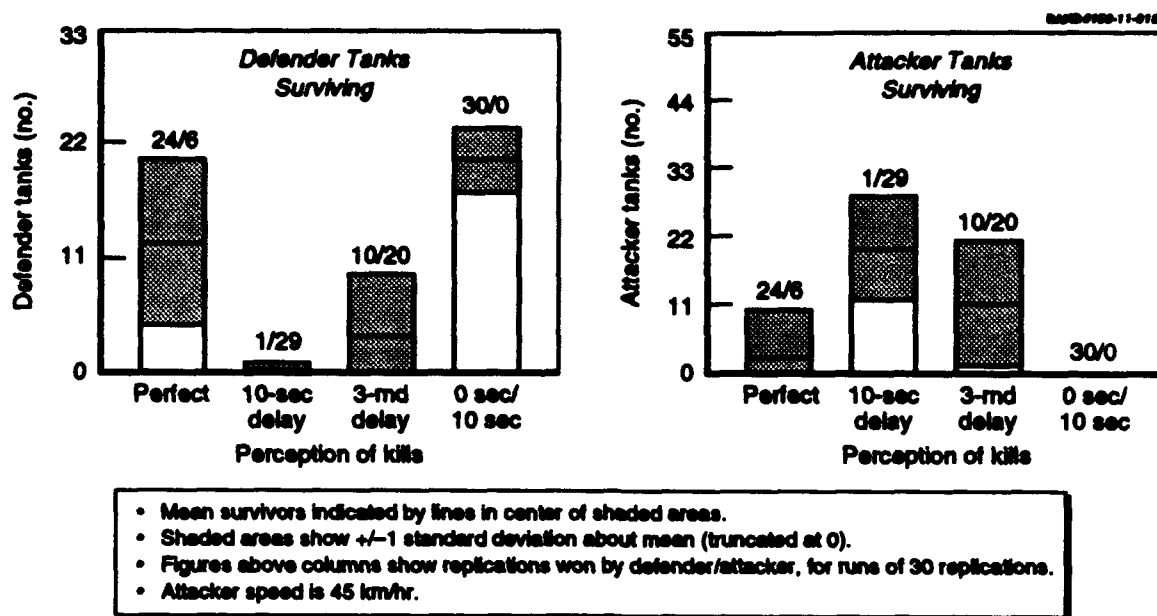


Figure 11—Stochastic Results: Effect of Varying Perception

The third case delays perception of kill not for a fixed time but until 3 hits have been recorded on a target. This case also favors the attacker, but not by as much as the 10-sec delay; several rounds can hit a target in less than 10 sec.

The final case depicted imposes a 10-sec delay on attacker perception of defender death, but no delay on defender perception of attacker death. This case, which may be more realistic, favors the defense.

Figure 12 summarizes the effect of changing the attacker's formation. All results assume perfect perception and a speed of advance of 45 km/hr. The first case shown is for the 2-up, 1-back formation, as in previous figures. The second case has 3 battalions (bn) in line, but the companies (co) within each battalion are in a 2-up, 1-back formation, as illustrated in Figure 6. As can be seen, this more concentrated attack favors the attacker, who wins in the majority of replications. The defender is less able to cope with one wave of attackers before the arrival of the next, and the attacker gets more of his systems across the gauntlet and into firing range. The third case, which puts all 9 attacker companies in line, is, as might be expected, even more favorable to the attacker.

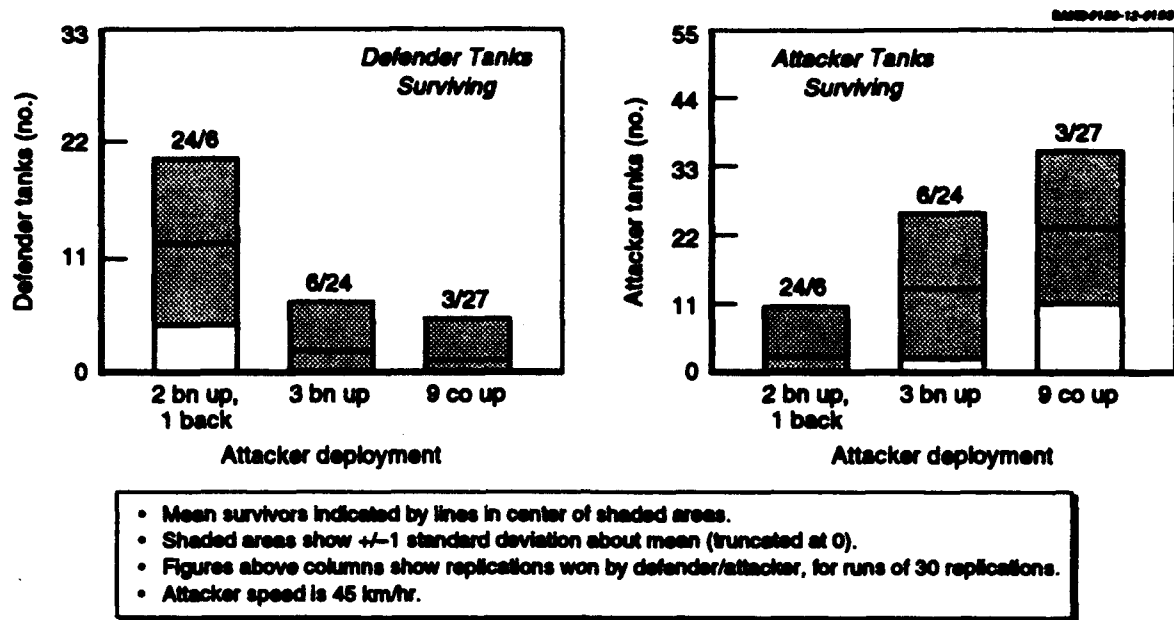


Figure 12—Stochastic Results: Effect of Deployment

4. AGGREGATE SIMULATION RESULTS

At this point we begin comparing results with a more aggregate model: a deterministic time-stepping simulation that calculates attrition rates between *groups* of weapon systems on each side. The model fights the battle through a succession of regular time steps. In each time step, group positions are updated, and hence the ranges between groups and the probabilities of kill change.

Each *group* directs all its fire at the closest living enemy *group* within range. There is no acquisition problem; all enemy groups are always visible. Perception is also perfect; dead groups cannot be mistaken for live ones.

In each time step, the kills by a group of its target group are calculated as the product of group rate of fire, duration of the time step, strength of the firing group at the start of the time step, and probability of kill at the range between the groups. If these calculations result in a group's taking losses exceeding its strength in a single time step, the shots fired and kills obtained by each enemy group firing at that group are reduced proportionately so that the total number killed is equal to the number of targets. Otherwise, there is no rounding; group strengths during the battle will generally not be integer. The calculation takes no account of projectile time of flight and assumes perfect distribution of all hits within a time step. In other words, all hits are on distinct targets within the target group—there is no overkill.

The model is implemented on the Microsoft Excel spreadsheet.¹

The size of groups can be varied in the model. The original scenario has been represented, and results will be shown at three levels of aggregation: company-sized groups, battalion-sized groups, and battalion vs. regiment. Figure 13 depicts these different levels of resolution.

Figure 14 shows the effects of attacker speed of advance in the deterministic model with company-size groups (3 defender groups vs. 9 attacker groups, as above). Also shown are the results obtained from the equivalent runs of the stochastic simulation. The label above the columns shows the winner in the deterministic model and the number of replications (out of 30 total) won by the defender or attacker in the stochastic model. The bar heights represent the mean number of survivors in the stochastic model. At low speeds of advance, the models are in good agreement: The defender wins with little or no loss. At

¹Anyone wishing to know more about this model should contact R. Hillestad or J. Owen at RAND.

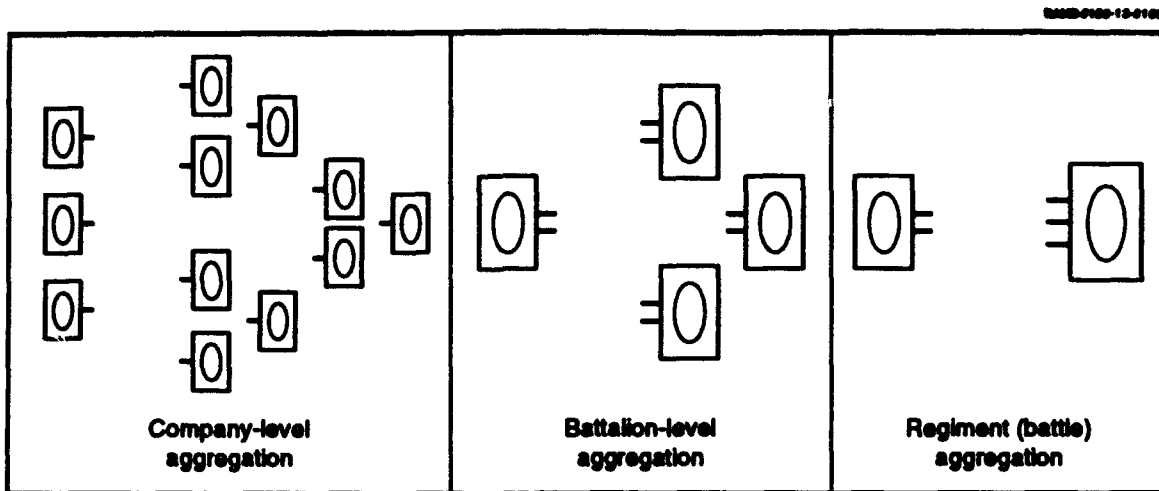


Figure 13—Levels of Resolution in the Aggregate, Deterministic Model

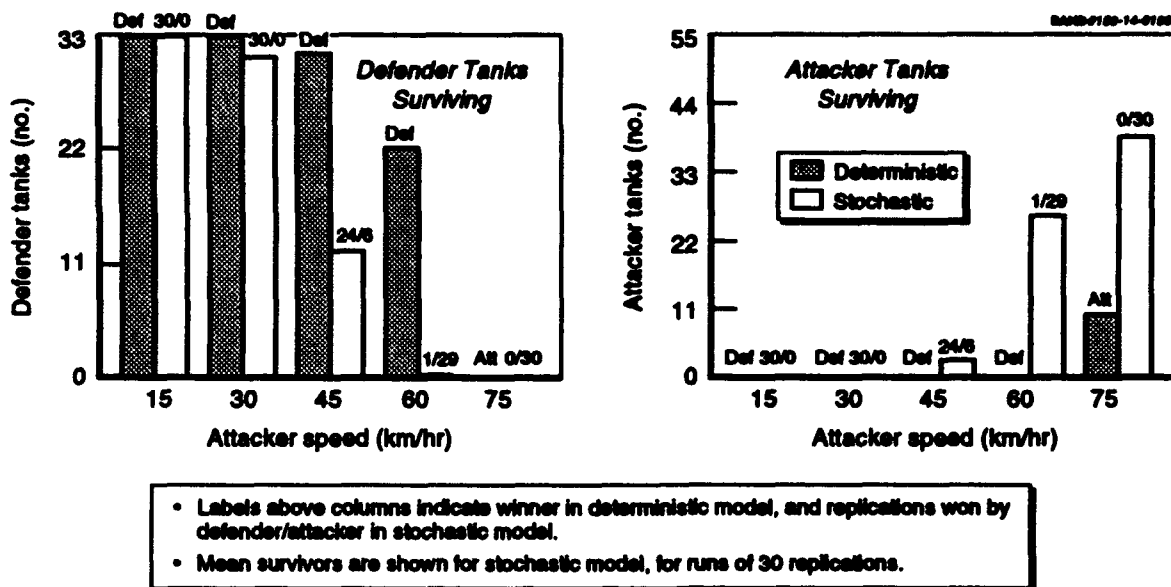


Figure 14—Stochastic and Deterministic Models: Effect of Attacker Speed

higher speeds, the deterministic simulation strongly favors the defender, as opposed to the stochastic model. Why is this?

In describing the deterministic model, we noted that it did not account for projectile time of flight. In the stochastic model, time of flight depends on range and projectile velocity;

at the ranges where the attacker suffers most losses it is typically about 3 sec. During that time, even with perfect perception of tank death, shots may be wasted by firing at targets that are about to be killed by shells already in flight. The deterministic model assumes that there are no multiple hits on the same target within a time step.

To test whether this difference in representation accounted for the difference in results, runs of the stochastic model were carried out in which the speed of projectiles was increased to a value at which time of flight was essentially zero. Figure 15 shows the deterministic-model results as before, but stochastic-model results with zero time of flight of shells. The latter are far closer to the results of the deterministic model.

This one difference in modeling led to most of the divergence in results between the two models in the cases examined. In constructing the aggregated model, an assumption was made that the time of flight could be left out of consideration without significantly influencing results (at least in the perfect-perception cases). That assumption turned out to be incorrect, illustrating the need to consider carefully and to test all the simplifications carried out in aggregation.

Setting the time of flight of projectiles to zero has brought the more detailed model's results closer to those of the more aggregated one. However, this is the wrong way 'round. The question is, How should time of flight, and the consequent effect on allocation of fire, be

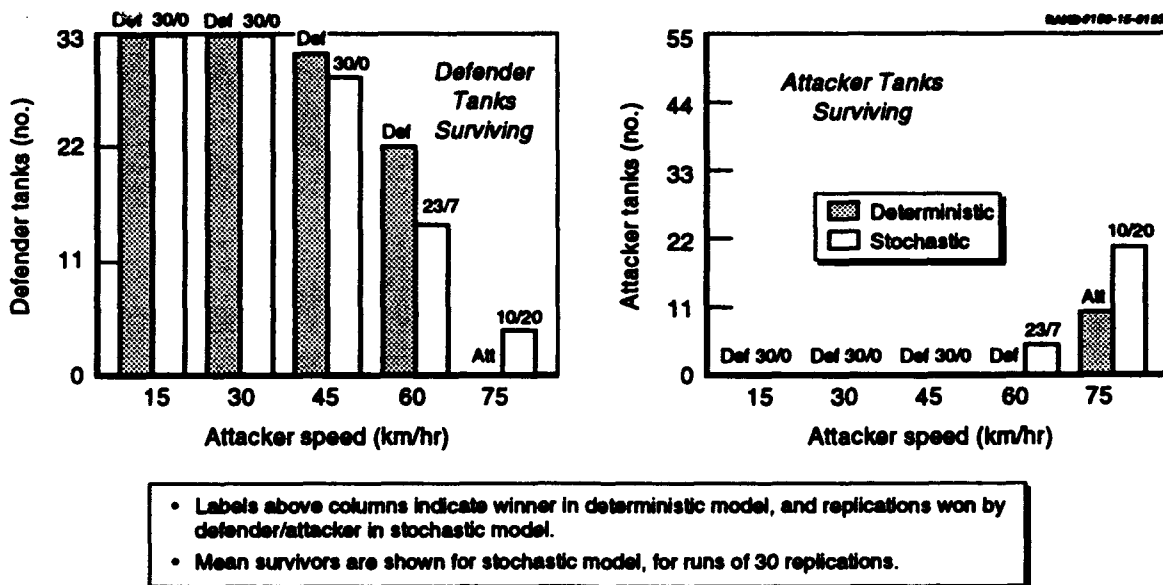


Figure 15—Stochastic- and Deterministic-Model Results: Zero Projectile Time of Flight

taken into account in the aggregated model? One approach is to determine an effective rate of fire for the deterministic model that matches that of the stochastic model. Figure 16 shows the results of varying the rate of fire in the deterministic, company-level model; the stochastic-model mean result is shown as the rightmost bar. Note that an effective rate of fire of 4 shots/min in the deterministic model most closely matches the results of the stochastic model in this case. This is an empirical result; there is no guarantee that it applies at other attack speeds, let alone more widely. We have not yet attempted to predict the result by side calculations.

The time step in a deterministic simulation is a form of aggregation in that the results of all processes that go on during a time step are computed at a single point in time using assumptions about the constancy of such rates as movement and firing. If the time step is too large, the state of resources may not change as fast as they should. That is, if we assume a certain number of weapons at the beginning of the step and compute the loss rate to the other side from that number of weapons, the loss may be exaggerated because some of the killing systems would have been destroyed themselves during the time step.

Figure 17 plots the number of defender tanks surviving in the original scenario at the end of runs with different speeds of attacker advance. All results are for a representation

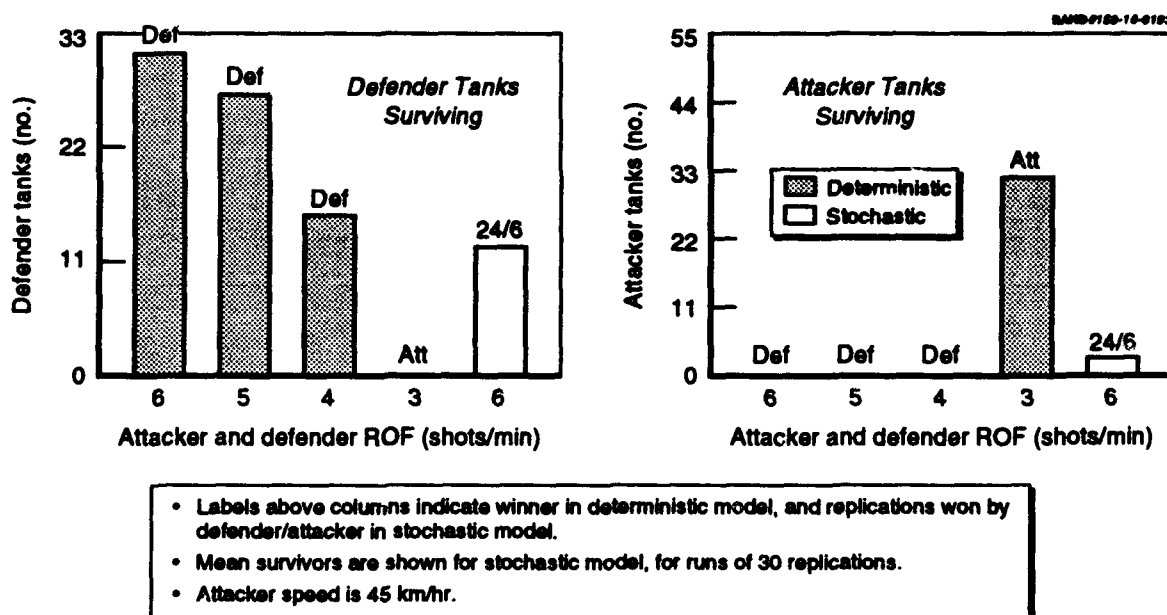


Figure 16—Comparison of Models: Varying the Effective Rate of Fire (ROF) in the Deterministic Model

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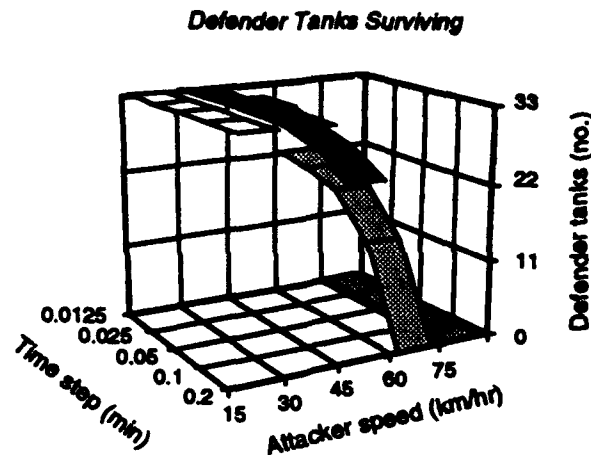


Figure 17—Deterministic Simulation: Effect of Time Step and Attacker Speed

with company-sized groups: 3 defender groups against 9 attacker groups. The third axis indicates length of time step used in the calculations. Twenty-five cases are shown here: all the combinations of 5 different time-step durations and 5 different speeds of advance. The results shown in the previous figures, which compared the deterministic- and stochastic-model results, are those with the shortest time-step value.

Increasing speed favors the attacker, as already noted. For the extreme speed values, length of time step has no effect on number of survivors. At the intermediate speeds, however, changing the time step changes the battle outcome dramatically: Long time steps favor the attacker. *Pks* in a time step are calculated using the ranges at the end of the time step: The shorter the range is, the less is the defender's advantage in lethality. For long time steps, the amount of time during which the attacker is unable to fire is reduced.

This example simply illustrates the importance of time-step length in this type of model. If this model were being used as the direct-fire attrition calculator in a corps or theater model, it might be convenient to have long time steps in order to limit run time; however, doing so could be dangerous. All subsequent illustrated results of this model will be those obtained with the shortest time-step value used here.

Consider next the size of the group represented in the deterministic model. What differences arise from using battalion- or regiment-sized groups as we make the model have even less (lower) resolution?

Figure 18 shows the number of defender and attacker tanks surviving at the end of the battle for different speeds of advance. The third axis varies the level of aggregation used in

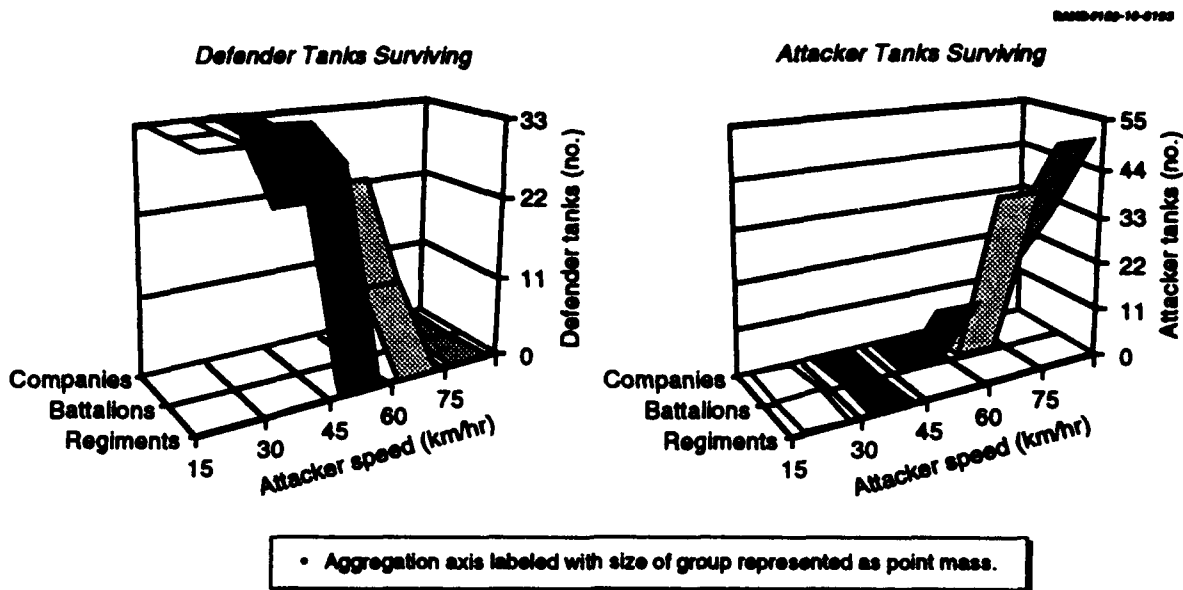


Figure 18—Aggregated, Deterministic Simulation: Effects of Level of Aggregation

the representation. The first set of results uses company-sized groups, as in previous figures. The second set pits 3 battalion-sized attacker groups against 1 battalion-sized defender group. The third set has a single regiment or brigade attacking group advancing on a battalion group.

The level of aggregation has no effect at low speeds: The attacker is unable to get into range with enough survivors to fire effectively, no matter what the level of aggregation. At higher speeds, the larger group sizes favor the attacker.

With company groups, the defender first engages the forward companies of the forward battalions: 4 in all, or four-ninths of the attacker's strength, then the rear companies of those battalions, 2 in all; then the 2 forward companies of the third battalion; and finally the third company of this battalion. It is also in this order that the attackers, if they survive, come into range to fire back.

With battalion groups, the forward 2 battalions, two-thirds of the attacker strength, come into range at the same time. With a single regimental group, the whole of the attacker's force comes into range at once. The effect of the greater aggregation is similar to that of putting more attacker companies into the front line. As shown in the stochastic model, the attacker does better with more units forward because he does not feed the units into the battle piecemeal and give the defender the advantage of shooting at only a few forward units at a time. However, the aggregation to large units *implies* that the entire

larger unit comes into range at once, which is an artifact of the aggregation, not an explicit assumption of the aggregate model.

Suppose it is desirable to represent a different configuration of the subunits in an aggregation. Is there a way to represent a different forward posture? Figure 19 shows the effects of attempting to represent the attacker's formation within a battalion-sized group by restricting the proportion of the group allowed to fire. Results are compared with those for company groups when the company groups are placed in the original 2-up, 1-back configuration. The graphs show defender and attacker survivors, at different speeds of advance. Results with company groups and battalion groups are shown as in the previous figure. The third and fourth sets of results also use battalion groups, but an extra factor was added to the calculation of attrition rates: a multiplying factor on the number of firers. This factor was left at 100 percent for the defender, but was set to 75 percent and 67 percent for the attacker's battalions, in the two sets of cases.

The factor has no effect at low speeds, at which the attacker gets little or no chance to fire. At higher speeds, an effect is apparent. Reducing the proportion of attacker allowed to fire favors the defenders, bringing the results back toward those with company groups. Indeed, if only two-thirds of the attackers can fire, the battle becomes more favorable to the defender than with company groups. It can be seen that with this level of attacker

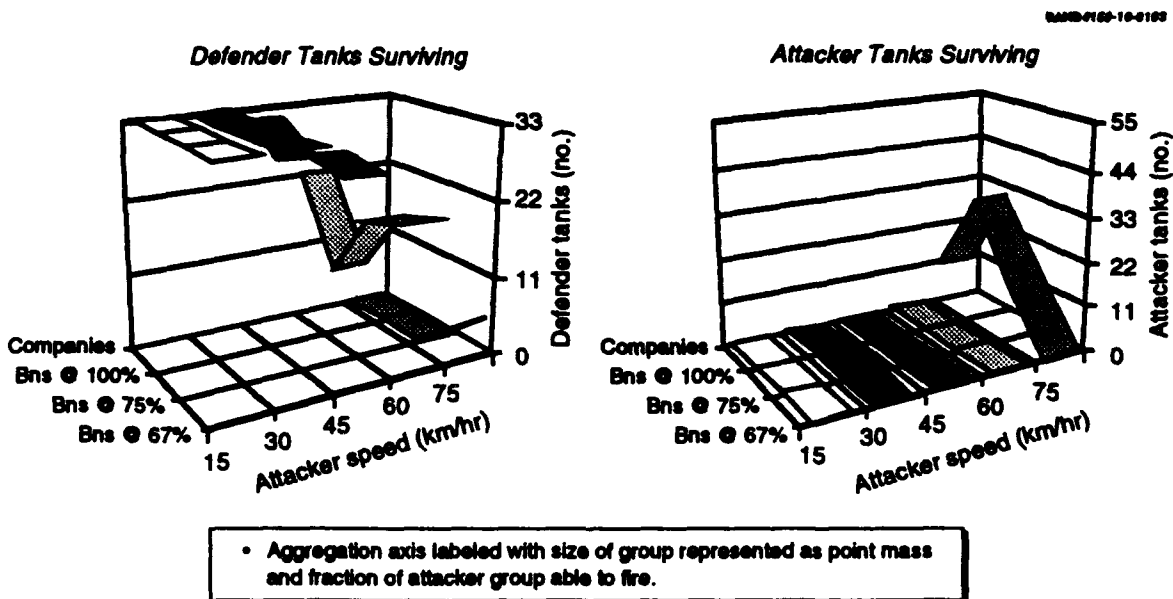


Figure 19—Aggregated Deterministic Simulation: Effect of Fraction Participating

participation, at the highest speed of advance used, the defender wins in that he has a few survivors at the end of the battle, and the attacker does not. A 75-percent level of participation seems to give a better approximation to the company-level results in the cases shown.

Thus, configuration can be represented when greater aggregation is used, but the modeling must be adjusted to do so. In this case an additional factor had to be added to the low-resolution, aggregate model. The value of that factor needs to be determined somehow, and it is probably quite situation dependent. In our case, it was done empirically. It is possible that it might be done predictively by careful consideration of the rates of advance and firing, and battle configuration. We have not attempted such a determination, but we note that others have not had much success.²

²For example, see Schaub, Thomas, *Zur Aggregation heterogener Abnutzungsprozesse in Gefechtssimulationsmodellen*, Institute for Systems Analysis and Operations Research, Der Universität Der Bundeswehr, München, 1991.

5. A CONSTANT-COEFFICIENT LANCHESTER MODEL¹

We also considered how well the battle described by the original scenario might be represented by a constant-coefficient (no time variation) square-law,² Lanchester model. For this basic case the familiar equations are

$$\frac{dx(t)}{dt} = -Ay(t) \quad (1)$$

$$\frac{dy(t)}{dt} = -Bx(t), \quad (2)$$

where $x(t)$ and $y(t)$ are the strengths of the two sides at time t , and A and B are the (constant) rates at which one unit of strength on one side causes attrition of the other side's strength. The well-known, closed-form solution for these equations is given in terms of hyperbolic functions as

$$x(t) = x_0 \cosh(\sqrt{AB}t) - y_0 \sqrt{A/B} \sinh(\sqrt{AB}t) \quad (3)$$

$$y(t) = y_0 \cosh(\sqrt{AB}t) - x_0 \sqrt{B/A} \sinh(\sqrt{AB}t), \quad (4)$$

where

$$x_0 = x(0) \quad (5)$$

$$y_0 = y(0). \quad (6)$$

The only problem is, What are the values to use for A and B , the attrition rate coefficients? If the initial strengths of the opposing forces are known, and also their

¹For another discussion on the use of this type of model in aggregation, see Hillestad and Juncosa, *Cutting Some Trees*.

²In this simple, one-weapon-system scenario, many weapons can fire at a given weapon of the other side at once—the situation for which the square law was formulated.

strengths at a given subsequent time, Lanchester coefficients that will give those same results at the end points can be calculated by the following formulas:

$$A = \frac{\sqrt{x_0^2 - x_f^2}}{t\sqrt{y_0^2 - y_f^2}} \ln(F) \quad (7)$$

$$B = \frac{\sqrt{y_0^2 - y_f^2}}{t\sqrt{x_0^2 - x_f^2}} \ln(F), \quad (8)$$

where

$$F = \frac{x_f\sqrt{y_0^2 - y_f^2} + y_f\sqrt{x_0^2 - x_f^2}}{x_0\sqrt{y_0^2 - y_f^2} + y_0\sqrt{x_0^2 - x_f^2}} \quad (9)$$

and

$$y_f = x(t_f). \quad (10)$$

$$y_f = y(t_f). \quad (11)$$

The time t_f is the end point at which the Lanchester system is to fit exactly to the simulation results. Therefore, given results from another model, such as the deterministic simulation, Lanchester coefficients can be found to give the same final result for the overall battle, starting from the same initial conditions. We now describe some results using this third model, with the coefficients derived from the deterministic simulation.

The graphs in Figure 20 show total defender and attacker strength over time in the battle, starting from when the defender is able to open fire. The attacker speed of advance is 45 km/hr, and the deterministic-simulation result shown used company groups. The second line on each graph shows strength using a constant-coefficient Lanchester square-law model.

For both attacker and defender, the constant-coefficient Lanchester model gives higher casualty rates in the early stages of the battle; in the simulation, lethalties start low and increase as the range closes. Both representations terminate at the same point, because the

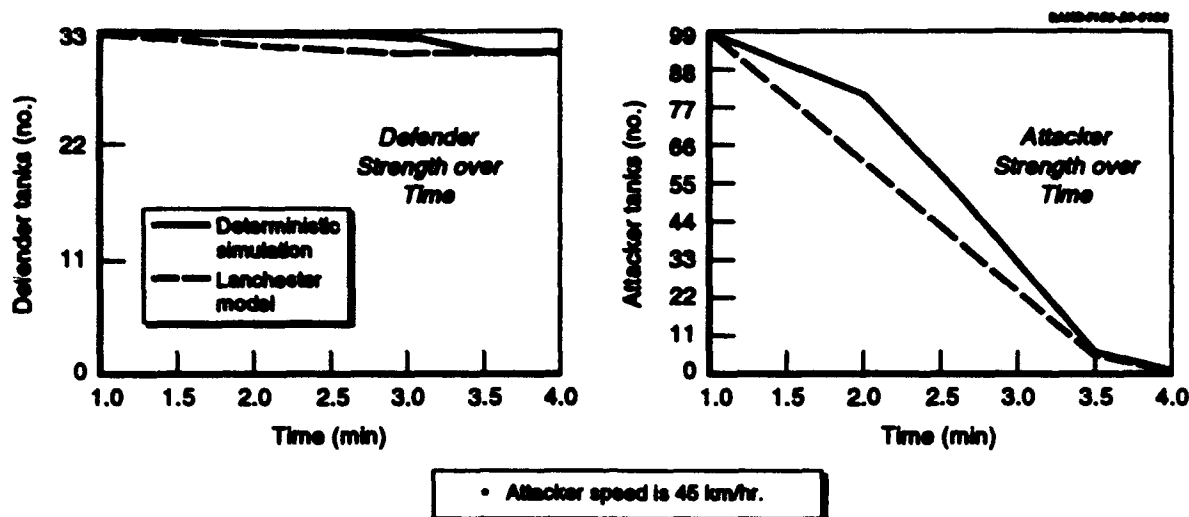


Figure 20—Constant-Coefficient Lanchester Results Fitted to the Company-Level Deterministic Simulation

constant coefficients were calculated to achieve this. Halfway through the battle, however, the difference in attacker losses is about 20 tanks, or 20 percent of initial strength. This difference might not matter if only the end result were to be used elsewhere. But if intermediate results are needed, or the battle was interrupted by the arrival of reinforcements, the differences may be important.

The differences become even more pronounced as one attempts to fit to the more aggregate deterministic simulation at the battalion and regiment level. Figure 21 shows the plots of Figure 20 but for a run of the deterministic simulation with battalion-sized groups. Similar effects are seen, but they are more significant for the defender than in Figure 20 because the deterministic simulation at this level of aggregation gives higher defender losses.

Figure 22 shows the equivalent plots, where the attacker is represented in the deterministic simulation by a single regiment-sized group. With this level of aggregation, the deterministic simulation gives an attacker victory. Similar differences between the simulation and the constant-coefficient Lanchester model at intermediate times can be seen, but they are even more pronounced. The constant-coefficient model does not represent the two stages of the battle: the first stage, in which the defender uses his range advantage and the attacker cannot fire; and the second stage, in which both sides can fire.

One option, given that it is easy in this scenario to define two distinct stages of the battle, is to approximate it by two constant-coefficient Lanchester battles rather than one. Figure 23 shows the results for the company-level aggregation again, but with an extra line

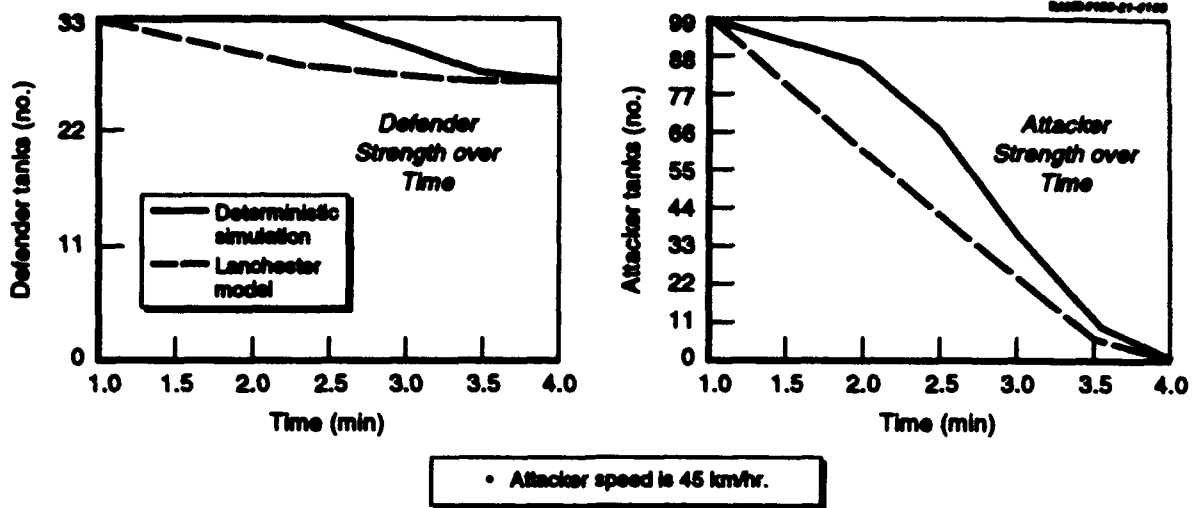


Figure 21—Constant-Coefficient Lanchester Results Fitted to the Battalion-Level Deterministic Simulation

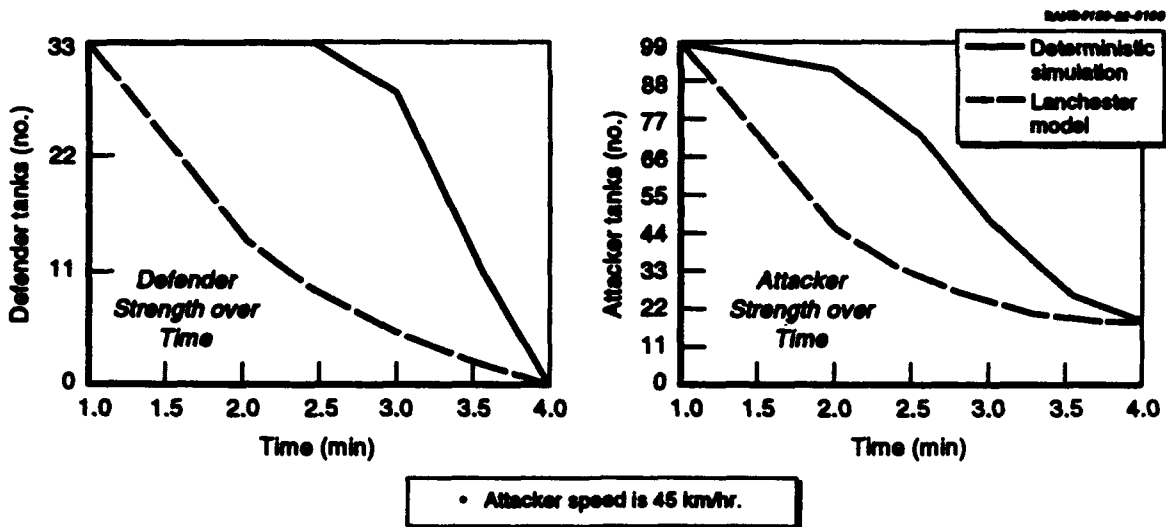


Figure 22—Constant-Coefficient Lanchester Results Fitted to the Regiment (Battle)-Level Deterministic Simulation

on each graph, showing the effect of using a two-stage constant fit. The first stage represents that part of the battle in which only the defender can fire; the defender's attrition coefficient is simply the number of attackers killed per defender per unit time, found by taking the attacker losses in the deterministic simulation at the end of this first stage of battle. The

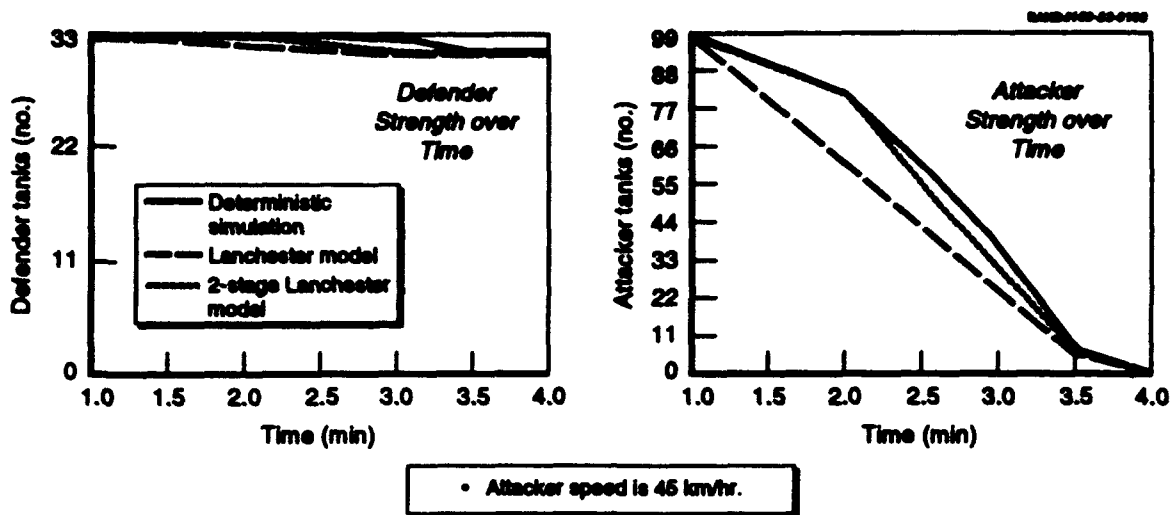


Figure 23—Fitting Two-Stage Constants to the Company-Level Deterministic Simulation

second stage is a constant-coefficient Lanchester battle, taking the position in the deterministic simulation at the end of the first stage as the initial strengths, the position at the end of the battle as the final strengths, and calculating Lanchester coefficients to fit. As can be seen, this battle produces a much better fit to the deterministic-simulation results.

Figures 24 and 25 show that the two-stage Lanchester approximation produces much better intermediate results for the battalion and regimental aggregations, as well. A common feature of the battles at the three levels of aggregation is that there are two distinct stages to the conflict, each with different characteristics. These stages must be included in a constant-coefficient representation if it is to produce a good approximation. Whether the deterministic-simulation results are themselves valid at the higher aggregations does not affect this argument.

Given an understanding of the nature of the battle, a simpler model fit can be found. This is encouraging in that such simple approximations could be used in place of more computationally intensive models to give essentially the same results. The problem, of course, is that of determining the coefficients without having to run the more detailed model. We have not yet attempted to determine how and whether the Lanchester coefficients could be obtained from knowledge of the battle parameters and configurations without using the detailed simulation.

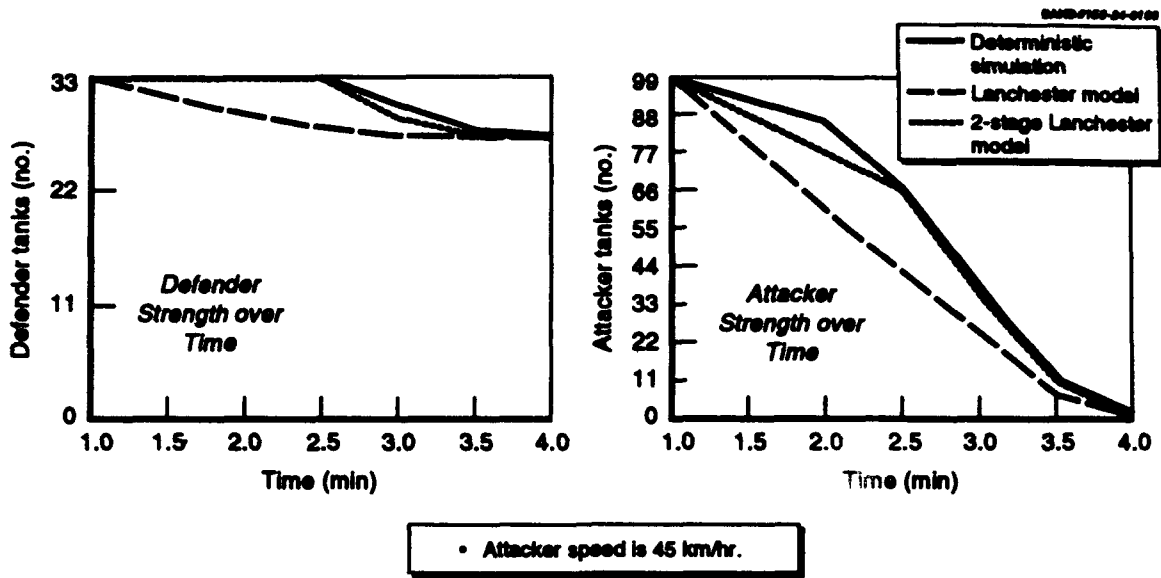


Figure 24—Fitting Two-Stage Constants to the Battalion-Level Deterministic Simulation

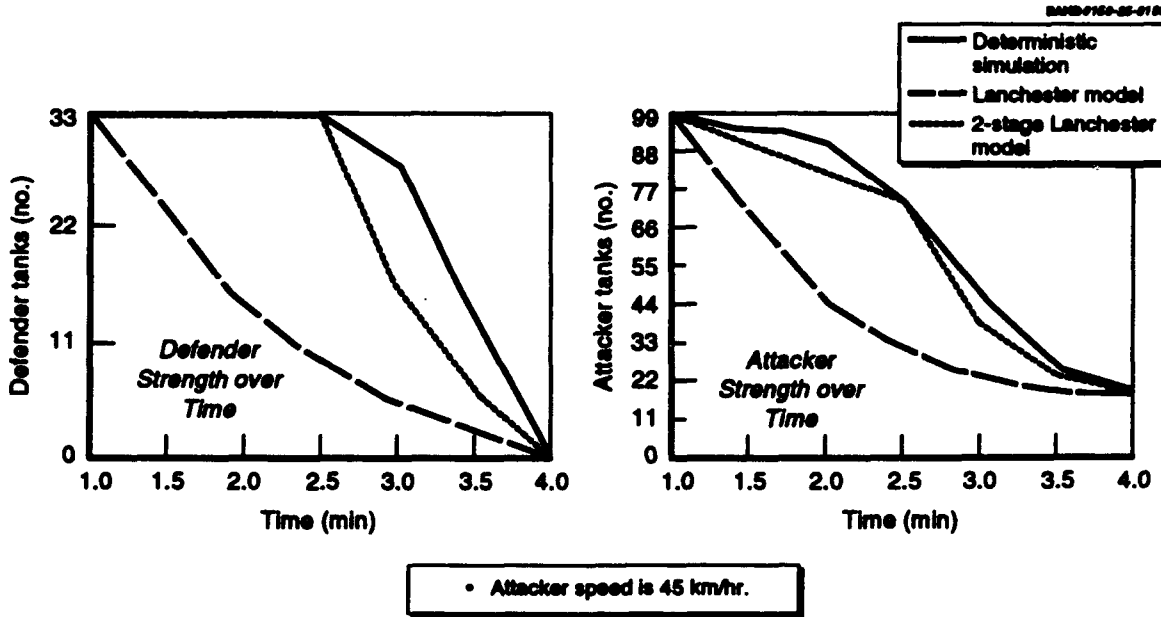


Figure 25—Fitting Two-Stage Constants to the Regiment-Level Deterministic Simulation

6. CONCLUSIONS

We have presented several sets of experimental results. What general points can be drawn? First, as observed in the examples from the detailed stochastic simulation, common intuition about outcomes, causes, and effects is frequently wrong. Since the battle configuration was 3 to 1 in favor of the attacker, one might have concluded, based on the 3:1 rule, that this should lead to a fair fight given some defender advantage (given, in our simulation, by the *Ph*-vs.-range advantage). However, the outcome was highly dependent on the attacker speed and deployment. Also, the effect of projectile time of flight caused a significant reduction in the effective rate of fire of the defender; when it was not taken into account in a deterministic simulation, the outcomes differed significantly.

The "fair-fight" conditions in which both defender and attacker won a significant number of battles also created results with the largest statistical variance. Furthermore, the battle outcomes were distributed bimodally; few cases terminated as a draw, even when the average indicated an even battle.¹ Such variance was not apparent in the deterministic model.

Unfortunately, considerable defense analysis is done in the fair-fight regime, and much of that analysis is done with deterministic models. This can mean that decision makers are not really provided analysis that uncovers the considerable uncertainties surrounding that analysis and their possible policy decisions. It should be possible to do better. For example, if one can define the probability of winning or losing with a deterministic model, the numbers of survivors in the two cases could be approximated by assuming distributions of the types found using stochastic models.

We showed that it is possible to scale results for different levels of resolution but that the scale factors are not obvious. One might suppose that the appropriate scale factor was 67 percent in the example given because the deployment was 2 up and 1 back. However, this assumption favored the defender unfairly relative to the more detailed results. In fact, a scale factor of 75 percent was more appropriate for the specific example given. We suspect that the factor is situation dependent but have not tested that fact. The conclusion we reached is that we do not yet know how to scale results.

Even simpler approximations, such as the constant-coefficient, square-law Lanchester model used by us, are possible. However, they, also, are situation dependent, and some

¹This was also observed in Hofmann, "On an Approach."

knowledge of the likely battle progression is needed. In fact, it is probably necessary to divide the battles into stages and have some time-varying component of the attrition equations to obtain an accurate representation. We do not know how easy it is to predict such stages or to predict the coefficients themselves from basic knowledge of the scenario, force capabilities, and deployments.

We observed another benefit of this exercise. One learns considerably more about both the problem and the models and modeling assumptions within them when more than one model is used on the same problem. For example, we observed the importance of the projectile time of flight on the results by comparing the stochastic model with the deterministic one. The effect of time of flight would have been inherent in the detailed stochastic results, but it would not have been apparent unless a set of cases were run in which it was varied. If the deterministic model only had been run, the importance of time of flight would have been overlooked. In some of the few defense-analysis efforts where different models have been used in parallel, the authors have seen the increase in understanding and insight into the problem. Despite possible additional costs of such a seemingly redundant approach to analysis, the potentially large differences in results, as demonstrated here, due to model aggregations, approaches, and assumptions would argue for some parallelism in approach.

Finally, it seems desirable to develop a broader effort of this nature across the defense-analysis-and-simulation community. We have only scratched the surface. One should investigate the effects of simulations with various aggregations of weapon types.² For example, how does one represent the different possibilities for fire allocation in a more aggregate model? Recently, using our detailed stochastic simulation with two weapon types with distinctly different characteristics, we showed that dramatically different results could be obtained with two "reasonable" (i.e., intuitively sensible) but different fire-allocation policies. How do terrain and line of sight affect the outcomes? One of the authors demonstrated that terrain which reduced the line of sight and firing opportunities shifted the balance toward the attacker because the attacker could get through the zone in which the defender had an advantage with fewer losses. This is contrary to the expectations of some that rough terrain would favor the defender. We were also able to show that when the defender advantage in range was removed and replaced by a "first-shot" advantage, the results shifted dramatically toward the attacker. Thus, further investigation should consider

²See Hillestad and Juncosa, *Cutting Some Trees*, for a theoretical discussion of this problem.

what the defender advantage really is—our scenario was only one hypothetical case—and how it could be represented in more aggregate models.

Such an organized effort, but on a larger scale, is both necessary and possible. Increasingly, simulation is being used and proposed within the defense community for training, extensions of testing, and other forms of analysis. The possibilities for bad training, wrong lessons, and bad analysis as a result of arbitrary aggregations and cross-coupling of models of differing resolution will increase, as well. At the same time, the proliferation of simulations distributes the problem of doing some organized military science with those simulations. It is hoped that the results described in this Note will spark some additional concern and interest within the community.